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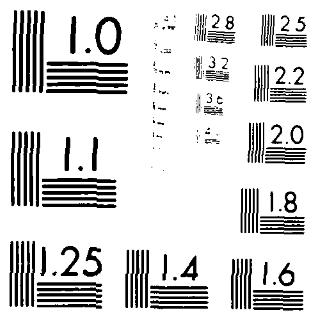

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NBDL-84R003

KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT TEST: CORRECTED FOR USE WITH "EEG-LIKE" DATA

M. S. Weiss

April 1984

NAVAL BIODYNAMICS LABORATORY
New Orleans, Louisiana

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The one-sample Kolmogorov-Smirnov goodness-of-fit test (KS) is designed for use with independent data and can be highly sensitive to correlated data. Standard critical values for KS cannot be used with data with known correlations. For data with EEG-like spectra (low frequency, high amplitude spectral peaks) an empirically derived correction for KS provides correct critical values and retains much of the power of the original KS. The correction is based on a simple quadratic expression involving a parameter, τ, computed from zero-crossing measurements.

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KOLMOGOROV-SMIRNOV GOODNESS-OF-FIT TEST: CORRECTED FOR USE WITH "EEG-LIKE" DATA

M. S. Weiss

April 1984

Naval Medical Research and Development Command
Research Work Unit No. M0097PN001-5004

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SUMMARY

THE PROBLEM

The standard Kolmogorov-Smirnov statistical test for goodness-of-fit (KS) requires that the data being tested come from independent samples. The use of this test with highly correlated time series data such as EEG data is inappropriate and yields erroneous results. A procedure is required to correct KS for the influence of correlation among the sampled data without sacrificing statistical power.

FINDINGS

For data with EEG-like spectra (low frequency, high amplitude spectral peaks) an empirically derived correction for KS provides correct critical values and retains much of the power of the original KS. The correction is based on a simple quadratic expression involving a parameter, τ, computed from zero crossing measurements.

Kolmogorov-Smirnov Goodness-of-Fit Test:
Corrected for use with "EEG-Like" Data

I. INTRODUCTION

Traditional methods of computerized time-series analysis (e.g., spectral analysis, period analysis) as well as more advanced multi-variate statistical analyses rely on assumptions, techniques and methodology developed for situations where the underlying statistics of the data to be analyzed are reasonably well understood. This is not true for electroencephalographic (EEG) time-series data where uncritical adoption of traditional approaches to the problem of EEG analysis may be inappropriate. The use of computer simulated EEG data provides a means of attacking this problem by quantitatively evaluating and comparing analytic procedures. It can also lead to the development of new procedures which are valid for time series data with properties similar to those of the EEG (EEG-like data).

The results reported here describe a modification of the Kolmogorov-Smirnov statistical test (KS) for normally distributed time series data when these data are highly correlated with spectral characteristics similar to those of EEG data. A general purpose mini-computer (DGC Eclipse® S/140) was used to generate and analyze simulated EEG-like time series using specially designed software (16, 17).

The KS for goodness-of-fit between a theoretical distribution and a sample set of observations is well known and widely used. It is defined as:

$$KS = \max|F(x)-S_n(x)| \tag{1.1}$$

where $F(x)$ is a population distribution function and $S_n(x)$ is the sample distribution step-function. For continuous $F(x)$ the sampling distribution of KS is known and is independent of $F(x)$. For the particular case of $F(x)$ normal with mean and variance estimated from the sample, Monte Carlo corrections for critical values of KS have been obtained (6, 12, 14). In the past decade there has been some attention devoted to applying KS to sampled EEG data (e.g. 7, 8, 13). These data, as most time-series data, can be highly correlated. Correct application of the KS (or any other statistic that demands independent data) requires that the effective sampling rate be reduced by discarding data in order to insure an uncorrelated ("white noise") sample (7). In practice, the original sample length is often of fixed duration so this procedure results in a decreased sample size and a consequent reduction in the power of the test. Ideally, an estimate of the correlation properties of the data could be used to correct the test statistic, eliminating the need for discarding data.

A general procedure using this approach has been developed and used to construct a corrected KS. This correction enables the use of all the data as originally sampled, increasing the power of the test. The results are applicable to EEG-like time series data and are an extension of work previously reported (14, 15).

II. METHOD

The simulated EEG time series were generated by linearly combining second-order autoregressive (AR) series. Though the autoregressive model has been widely used and extensively developed (1, 3), some pertinent ideas and results are outlined here. A time series, which is a set of sequential observations of some (stochastic) process, is said to be autoregressive of order p if each observation, X_t can be expressed as:

$$X_t = a_1 X_{t-t_s} + a_2 X_{t-2t_s} + \cdots\cdots + a_p X_{t-pt_s} + a_{p+1} E_t \qquad (2.1)$$

where each E_t is an independent sample from a zero mean, unit variance, random process with an arbitrary distribution function and t_s is the time between the successive observations X_{t-t_s} and X_t. The second-order AR process is of particular theoretical and practical importance and can be expressed as:

$$X_t = a_1 X_{t-t_s} + a_2 X_{t-2t_s} + a_3 E_t \qquad (2.2)$$

with the following stability conditions required for the process described by (2.2) to remain bounded:

$$\begin{aligned} |a_2| &< 1 \\ |a_1| &< 1-a_2 \end{aligned} \qquad (2.3)$$

The mean of X_t is zero and:

$$\sigma_X^2 = a_3^2(1-a_2)/[\{(1-a_2)^2 - a_1^2\}\ (1+a_2)] \qquad (2.4)$$

The normalized autocorrelation function corresponding to (2.2), for $a_1^2 + 4a_2 < 0$, is:

$$P_k = (-a_2)^{|k|/2} \cos(\omega_0 k - \phi_0)/\cos \phi_0) \quad k = 0, \pm 1, \pm 2, \ldots\ldots \qquad (2.5)$$

where: $\cos \omega_0 = a_1/2(-a_2)^{1/2}$

$$\tan \phi_0 = \frac{1+a_2}{1-a_2} \tan \omega_0$$

The corresponding expression for the one-sided, power spectral density function is:

$$P(f) = 2t_s a_3^{\,2}/D(f)$$

where $D(f) = 1 + a_1^{\,2} + a_2^{\,2} - 2a_1(1-a_2)\cos 2\pi f t_s - 2a_2\cos 4\pi f t_s$ (2.6)

The autocorrelation functions described in (2.5) consist of damped sine waves and exponentials and consequently can be used to describe a wide variety of natural phenomena, including EEG activity (2, 4, 9). In addition, examination of (2.6) shows that for $|a_1(1-a_2)| < |4a_2|$ and $a_2 < 0$, the spectrum will have a peak at ω_0 if $\cos \omega_0 t_s = a_1(a_2-1)/4a_2$. Thus processes such as EEG activity, which have peaked spectra, can often be approximately represented by a combination of second-order AR series. The simplest procedure is to use an independent linear combination of such series with each series selected to represent one of the desired spectral peaks. By appropriately selecting each a_3 (2.2) the relative amplitude of each peak can be specified. The resultant time series for up to five independent peaks is represented by the sum:

$$s_t = \sum_{n=1}^{N} a_{1_n} X_{t_n - t_s} + a_{2_n} X_{t_s - 2t_s} + a_{3_n} E_{t_n} \qquad 1 \le N \le 5 \qquad (2.7)$$

The corresponding spectrum is simply:

$$F(f) = \sum_{n=1}^{N} P_n(f) \qquad 1 \le N \le 5 \qquad (2.8)$$

which is a linear combination of spectra of the form in (1.6), with the relative weights determined by appropriate selection of a_{3_n}.

The actual procedures involved selecting the location, relative amplitude and bandwidth (f_n, r_n, b_n) for each peak. The first two parameters in (2.7) were chosen by computing:

$$\begin{aligned} a_{2_n} &= .1b_n - 1 \\ a_{1_n} &= 4a_{2_n}\cos f_n/(a_{2_n} - 1) \end{aligned} \qquad (2.9)$$

A more precise form for the half-power bandwidth is $a_{2_n} = .06b_n - .9$

The third parameter, a_{n} was computed so that the total variance of the resultant time series (2.7) was r_n and the amplitude of (2.7) at each peak was proportional to r_n:

$$a_{n} = K_n r_n D_n(f_n) \tag{2.10}$$

where

$$K_n = \sum_{n=1}^{N} r_n / \sum_{n=1}^{N} (r_n D_n(f_n)((1-a_{.})^2-a_{.}^2)(1+a_{.})/(1-a_{.}))$$

Equations (2.7) - (2.10) were used to simulate a variety of EEGs using both normal and non-normal distributions for E_t. The details of the basic Monte Carlo procedure used in these simulations, are described elsewhere (14, 16). The parameters for each of these time-series are detailed in Table 1. These simulated data were then used to compute empirical KS statistics. Previous results (14, 15) showed that for normally distributed E_t there was no simple relationship between the empirical KS statistic (KS_e) and the autoregressive parameters in (2.7). However, series with narrow low frequency peaks in the spectra (2.8) yielded larger KS_e values than series with broader or higher frequency spectral peaks. Two spectral parameters reflected this effect: m_2, the square root of the second moment, and m_4, the fourth root of the fourth moment of (2.8). In particular, the difference $1/m_2 - 1/m_4$ was monotonically related to KS_e. Using this result, a dimensionless parameter τ was defined as:

$$\tau = (1/m_2 - 1/m_4)/t_s \tag{2.11}$$

For a given set of simulated EEG samples, the population spectral moments and τ can be computed directly from (2.8) by integration. In actual practice, however, this is not possible since the population parameters are unknown and estimates must made from sample parameters. An alternative to using sample spectral estimates for computing τ lies in the zero crossing approach. If Z_n is the average zero crossing rate of the nth derivative of a time series and P(f) is the power spectral density of the time series then the following holds (10):

$$(Z_n/2)^2 = \frac{\int_0^\infty f^{2(n+1)} P(f)\, df}{\int_0^\infty f^{2n} P(f)\, df} \tag{2.12}$$

By definition, the moments of $P(f)$ are:

$$M_{2n} = \frac{\int_{-\infty}^{\infty} f^{2n} P(f)\, df}{\int_{-\infty}^{\infty} P(f)\, df} \qquad n = 0,1,2,\ldots\ldots \tag{2.13}$$

and $M_{2n+1} = 0$ since $P(f)$ is an even function.

Combining (2.12) and (2.13):

$$\frac{M_{2(n+1)}}{M_{2n}} = (Z_n/2) \;;\quad M_0 = 1 \qquad n = 0,1,2,\ldots\ldots \tag{2.14}$$

which can be rewritten as:

$$M_{2n} = \prod_{i=0}^{n-1} (Z_i/2) \qquad n = 1,2,3,\ldots\ldots \tag{2.15}$$

This final relationship (2.15) expresses the moments of the power spectral density as a product of the average *zero crossing* rates of the derivatives of the original time series. In particular, for the second and fourth moments:

$$\begin{aligned} M_2 &= (Z_0/2)^2 \\ M_4 &= (Z_0 Z_1/4)^2 \end{aligned} \tag{2.16}$$

Therefore:

$$m_2 = Z_0/2$$

$$m_4 = (Z_0 Z_1)^{1/2}/2$$

and:

$$\tau^2 = ((2/Z_0 - 2/(Z_0 Z_1)^{1/2})/t_s \tag{2.17}$$

The general correction for KS_e takes the form

$$KS_c(\alpha,n) = KS_e(\alpha,n)/G(\alpha,n,\tau) \tag{2.18}$$

where n is the sample size, α is the significant level, KS_e is the measured KS and KS_c is the corrected KS. KS_c has the same distribution as the tabulated KS which is based on independent samples. The general properties of G are such that G approaches 1.0 for $\tau<1.0$ and increases montonically with increasing τ. Rewriting (2.18) as:

$$G(.05,n,\tau_\varepsilon) = \frac{KS_e(.05,n)}{KS_c(.05,n)} \quad (2.19)$$

where τ_ε represents the empirical estimate of τ using sample zero crossing rates in (2.17), the problem is to find an appropriate analytic form for G. The solution to this problem is the outcome of the results reported below.

III. RESULTS

Ten thousand samples of seven different normally distributed simulated EEGs (2.7) with n ranging from 32 to 512 and τ (2.11) ranging from 1.03 to 2.80 were generated. $KS_e(\alpha,n)$ was computed for $\alpha = .05$. An additional set of samples was generated for an uncorrelated time-series (EEG type 8). Table 1 lists the simulated EEG parameters and Figure 1 illustrates the spectra and autocorrelation functions of each of the seven EEG types. A least-mean-squares fit for the following functional forms of G (2.19) was computed, using the simulated results as input to the BMD program for derivative-free non-li- regression (BMDPAR):

$$\text{Polynomial:} \quad \sum_{i=1}^{3} (P_{1_i} + P_{2_i}/n + P_{3_i}/n^2)\tau_\varepsilon^{\,i-1}$$

$$\text{Exponential:} \quad 1 + (P_1 + P_2/n^j)\tau_\varepsilon e^{-\tau_\varepsilon(P_3 + P_4/n^k)} \qquad j = 1,2;\ k = 1,2$$

The results of this least-mean-squares fit were tested using a second independent set of simulated EEGs. The final form chosen for G was:

$$G = 1.0 + 0.3/n + (.09 - 1.6/n)\tau_\varepsilon^{\,2} \quad (3.1)$$

which minimized residuals and had the fewest parameters.

This contrasts with previous results using simpler time series which suggested a linear function of τ (14). Table 2 summarizes the results of the second set of simulations which led to the selection of (3.1). This table indicates, as expected, that the use (or more appropriately, misuse) of the uncorrected KS_e statistic with correlated data leads to a high probability of falsely rejecting the true hypothesis of an underlying normal distribution (Type I error). Application of the correction formula and use of the corrected KS_c statistic reduces the probability of Type I error, i.e. the significance level, to the appropriate range of values.

The correction equations, (3.1) and (2.18), were then applied to a set of non-normally distributed data. Three unimodal non-normal distributions were used to generate E_t (2.7) and the resulting distributions for each of the simulated data sets are illustrated in Figure 2 where the vertical axes indicate percentages. The bottom row of this figure (EEG type 8) shows the underlying histograms for the three types of distributions. Distributions A and B are peaked while C is truncated and skewed. Sample time series, corresponding to each of these distributions are illustrated in Figure 3.

For each type of simulated EEG 10,000 samples were generated, for a total of 21 sample sets. The power of the modified KS statistic KS_c, was measured at both the .01 and .05 significance levels and and is summarized in Table 3. The uncorrected statistic would, of course, yield an artificially greater power in all cases due to a larger α. The probability of a Type II error (accepting the false hypothesis of a normal distribution) is 1 - power and obviously depends on the true underlying distribution. In all cases, power increases with increasing sample size.

IV. DISCUSSION

For correlated data with EEG-like spectra with strong peaks in the lower portion of the spectrum KS as a test for normality can be used by applying equations (2.17) -(3.1). An alternative is to decimate the data by decreasing the effective sample rate, yielding an uncorrelated series. The problem with this approach is that there is no a priori way to decide how much data to discard. For fixed length samples this may result in very small samples with a corresponding reduction in power. For the present data an attempt to address this problem resulted in Table 4. Normal samples of simulated EEG were generated with sample size equal to 512. Sub-samples with every second, fourth, eight and sixteenth point selected were then analyzed by computing KS_c and KS_e as described above. When KS_e approached the correct 1% and 5% critical values, the time-series could be considered effectively uncorrelated. Depending on the EEG type this could mean selecting anywhere from every fourth to every sixteenth point. Since in practical situations sample size is kept to a minimum to maximize stationarity, this would not be a useful approach. For example, taking a sample length of 1 second (7) would mean 60 to 100 points at the Nyquist sampling rate. Reducing this by a factor of four to sixteen would result in sample sizes on the order of six to 25, clearly too small.

It might be possible to develop an optimal decimation procedure which would retain the power of KS for uncorrelated data. Similarly, spectral or correlation properties other than τ might yield improved correction techniques. These remain problems for further research. The current solution is an easily implemented and computationally efficient method for correcting the Kolmogorov-Smirnov one sample test for goodness-of-fit.

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TABLE 1

Parameters of Simulated EEG

EEG Type	a_1	a_2	a_3	τ
1	1.78	-.80	.062	
	1.04	-.88	.55	
	-.49	-.73	.32	
				1.03
2	1.78	-.80	.088	
	1.04	-.88	.56	
	-.49	-.73	.32	
				1.15
3	1.78	.80	.087	
	1.02	-.85	.35	
	-.53	-.85	.18	
				1.46
4	1.50	-.6	.25	
	1.20	-.6	.53	
				1.78
5	1.65	-.7	.15	
	1.37	-.7	.39	
				2.10
6	1.78	-.8	.09	
	.99	-.8	.21	
				2.45
7	1.78	-.8	.09	
	.95	-.75	.19	
				2.80
8	0.0	0.0	1.0	
				0.69

TABLE 1: This table list the autoregressive coefficients and the computed value for τ for the time-series generated by equation (2.7).

TABLE 2

Empirical significance levels for corrected and uncorrected KS

EEG	KS	N: 32, α: 1.0	32, 5.0	64, 1.0	64, 5.0	128, 1.0	128, 5.0	256, 1.0	256, 5.0	512, 1.0	512, 5.0
1	COR	.7	3.8	1.0	5.2	1.0	5.2	1.6	5.8	1.8	6.5
	UNC	1.4	5.9	2.3	8.3	2.3	8.3	3.2	9.8	3.4	10.7
2	COR	.9	3.9	1.1	5.4	1.1	5.4	1.4	6.7	1.8	7.0
	UNC	1.7	6.5	2.9	9.8	2.9	9.8	4.3	12.3	4.6	13.5
3	COR	.8	3.8	1.0	4.2	1.0	4.2	1.3	5.1	1.6	5.3
	UNC	2.1	7.6	3.9	11.1	3.9	11.1	6.1	16.2	6.7	16.7
4	COR	1.1	4.8	.9	3.9	.9	3.9	.7	3.5	.8	3.8
	UNC	4.3	12.6	4.8	13.5	4.8	13.5	5.9	16.0	6.4	16.6
5	COR	1.4	5.5	1.1	4.6	1.1	4.6	1.1	3.8	1.1	4.0
	UNC	6.8	17.2	8.4	20.0	8.4	20.0	10.1	23.2	10.1	23.2
6	COR	.9	3.4	1.0	3.8	1.0	3.8	2.3	6.5	2.4	6.9
	UNC	7.5	18.1	14.5	28.4	14.5	28.4	23.8	39.7	26.4	43.2
7	COR	.8	3.5	1.1	3.7	1.1	3.7	2.5	6.0	2.1	6.[illegible]
	UNC	10.4	22.5	20.3	35.3	20.3	35.3	31.4	48.3	33.4	50.9
8	COR	.7	3.5	.6	3.2	.6	3.2	.7	3.3	.8	3.6
	UNC	1.14	4.9	1.0	4.7	1.0	4.7	1.2	5.0	1.33	5.3

TABLE 2: Critical values (%) computed for KS_c (COR) and KS_e (UNC) using equations (3.1) and (2.19)

TABLE 3

Power (%) of corrected KS statistic for non-normal time-series

EEG	N	A α: 1.0	A α: 5.0	B 1.0	B 5.0	C 1.0	C 5.0
1	32	3.1	8.0	1.4	5.3	1.0	4.8
	64	8.4	16.5	2.9	8.1	2.7	8.3
	128	20.9	31.3	5.6	12.8	4.4	12.1
	256	39.9	52.0	10.0	18.8	7.7	18.1
	512	65.0	75.8	17.0	28.5	15.4	29.8
2	32	2.9	8.2	1.6	6.0	1.5	5.8
	64	8.7	17.1	3.0	8.4	2.4	7.9
	128	20.3	30.3	6.0	13.2	4.9	12.5
	256	39.4	51.1	9.6	18.6	8.2	18.5
	512	63.5	74.3	16.1	27.3	15.0	29.9
3	32	3.0	7.6	1.3	5.1	1.5	5.5
	64	8.0	15.5	2.6	7.5	2.3	6.9
	128	21.1	30.2	6.0	12.6	4.7	11.8
	256	37.4	48.3	9.7	18.0	9.8	19.7
	512	60.5	70.9	14.8	24.7	19.3	34.3
4	32	10.5	19.8	4.4	10.9†	5.5	14.3
	64	20.7	31.4	6.8	13.7†	9.2	20.7
	128	35.2	47.3	9.9	17.7	18.9	35.5
	256	57.8	69.2	16.1	26.5	41.4	62.5
	512	84.2	91.2	25.7	39.3	77.5	90.4
5	32	9.0	17.6	4.0	10.6	4.5	12.6
	64	14.6	24.0	5.3	12.0	6.2	15.0
	128	28.0	39.1	7.8	14.8	10.8	22.6
	256	47.5	58.9	11.8	20.2	22.3	40.5
	512	72.0	81.9	17.7	28.9	48.6	68.5
6	32	3.1	8.4	2.0	5.7	1.7	5.7
	64	6.3	12.6	2.6	6.5	2.2	6.7
	128	16.9	25.4	6.5	12.2	5.8	12.5
	256	32.9	43.0	10.7	17.7	10.9	20.8
	512	54.1	64.2	14.9	23.4	21.5	36.4
7	32	3.8	8.7	1.8	5.5	1.6	5.2
	64	5.4	11.1	2.4	6.3	2.1	6.2
	128	14.8	22.7	5.7	11.2	5.2	11.4
	256	29.7	38.4	9.3	15.7	9.7	17.7
	512	48.9	58.3	12.8	20.9	18.6	32.5
8	32	43.9	58.4	18.3	32.3	92.7	98.4
	64	71.5	83.5	34.3	51.3	100.	100.
	128	94.5	98.0	62.8	78.9	100.	100.
	256	99.9	100.	91.0	96.9	100.	100.
	512	100.	100.	99.8	100.	100.	100.

† These entries were replicated once with the following results:
N=32, 4.4%, 10.9%; N=64, 5.6%, 12.3%

TABLE 4

Empirical significance levels (%) for corrected and uncorrected KS using decimated data

	N:	256		128		64		32	
EEG	KS / α:	1.0	5.0	1.0	5.0	1.0	5.0	1.0	5.0
1	COR	1.7	6.1	.7	3.7	.7	3.6	.7	3.8
	UNC	2.2	7.5	1.0	4.9	1.1	4.9	1.0	5.0
2	COR	1.5	6.1	.8	3.8	.7	3.4	.7	3.7
	UNC	2.5	8.0	1.3	5.4	1.2	4.8	1.0	4.7
3	COR	1.0	4.2	.5	2.7	.5	2.9	.8	3.8
	UNC	2.6	8.9	1.3	5.6	1.1	4.9	1.2	5.0
4	COR	.4	2.7	.5	3.1	.7	3.3	.7	3.8
	UNC	1.5	6.2	1.0	4.6	1.1	4.9	1.2	4.9
5	COR	.6	2.9	.6	2.8	.7	3.4	.7	3.8
	UNC	2.3	8.4	1.2	4.9	1.2	5.1	1.1	5.0
6	COR	1.1	4.0	.4	2.2	.3	2.5	.7	3.2
	UNC	8.5	20.5	2.6	9.0	1.2	5.6	1.0	4.5
7	COR	1.1	3.8	.5	3.6	.4	2.8	.5	3.2
	UNC	12.1	25.2	2.5	11.0	1.5	5.9	1.0	4.5
8	COR	.9	4.0	.7	3.4	.6	3.3	.7	3.7
	UNC	1.6	5.9	1.2	4.8	1.0	4.8	1.0	5.0

TABLE 4: Values computed for KS_c (COR) and KS_e (UNC) using normal samples of size 512 with every second, fourth, eighth and sixteenth point selected for computation.

NORMALIZED SPECTRAL DENSITY NORMALIZED AUTOCORRELATION

EEG TYPE

1

2

3

4

5

6

7

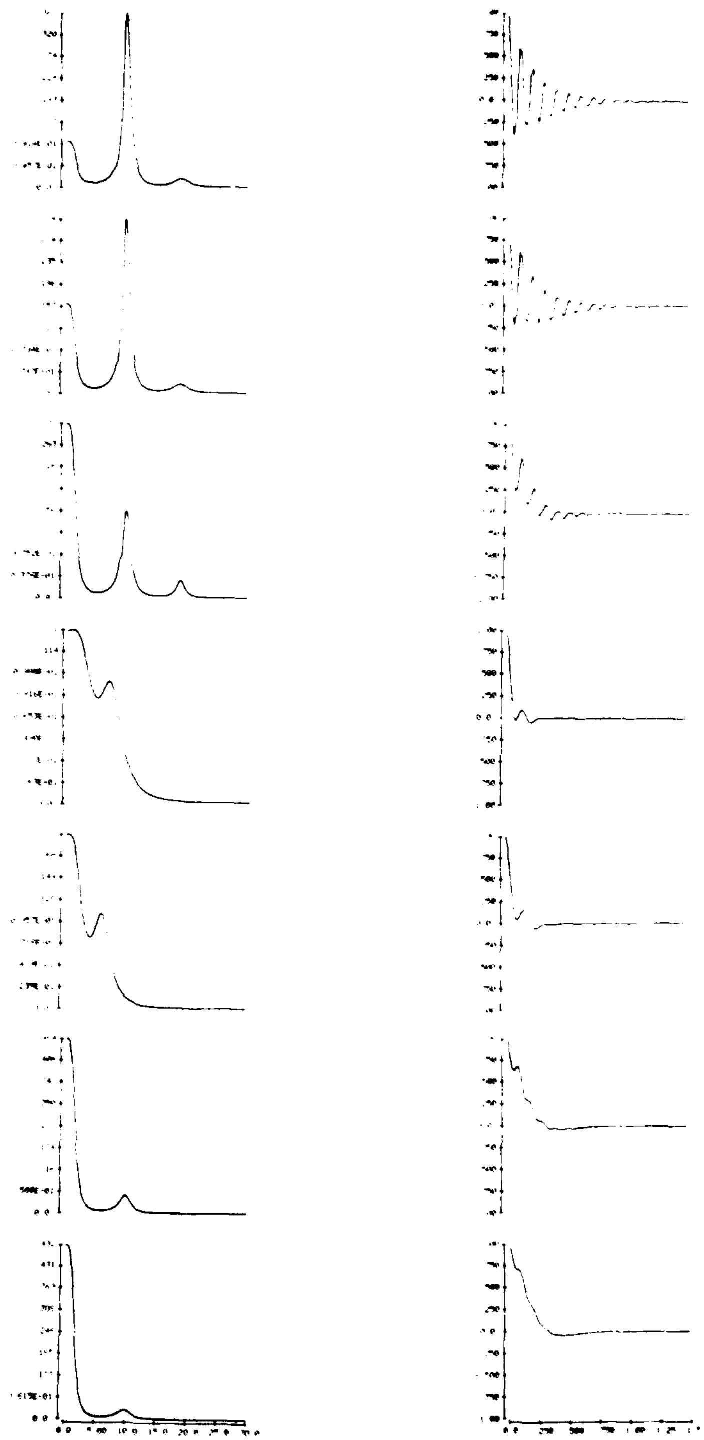

FIGURE 1 SPECTRAL DENSITY AND AUTOCORRELATION FUNCTIONS FOR SIMULATED EEG'S.

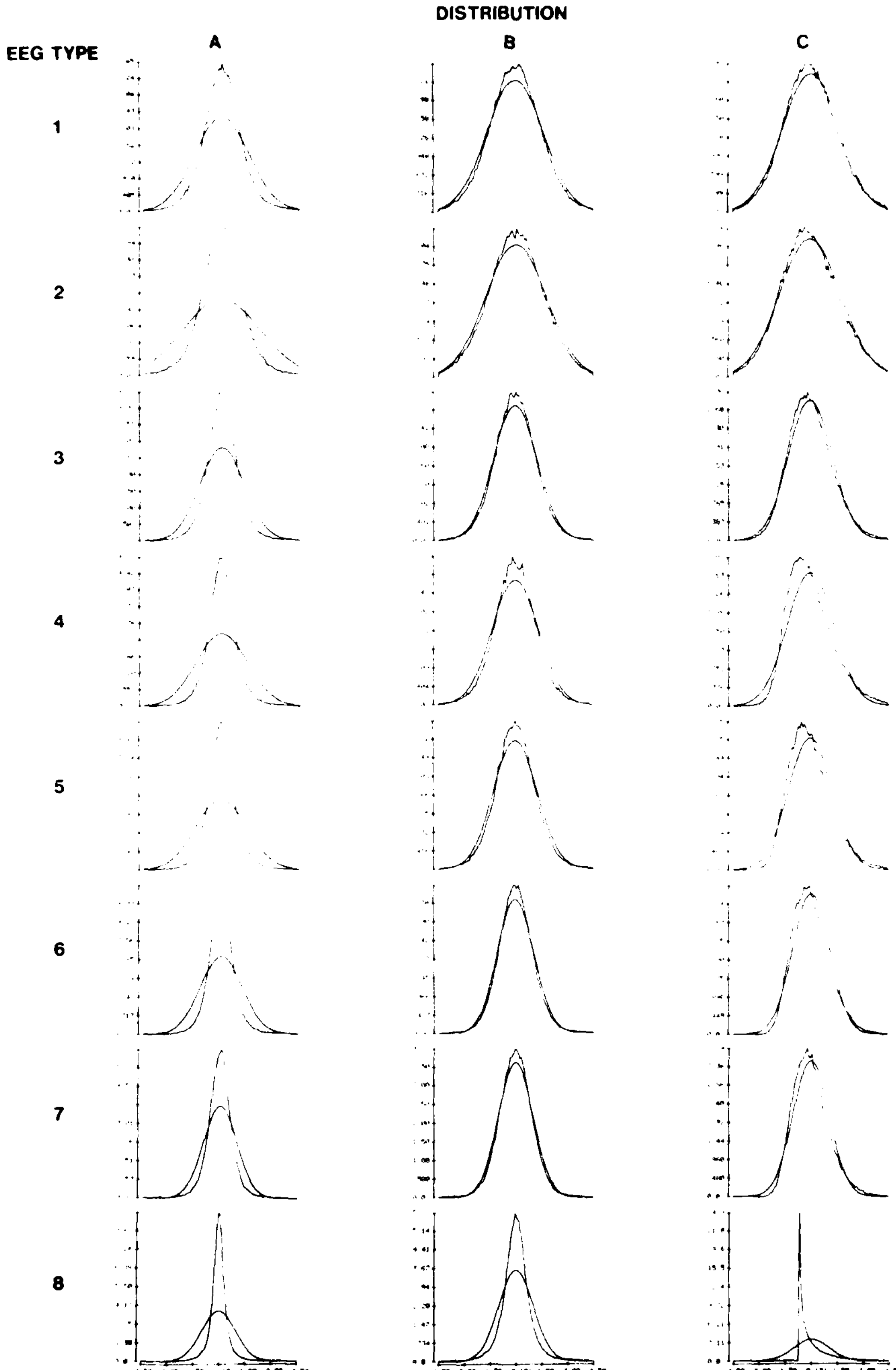

FIGURE 2 AMPLITUDE HISTOGRAMS FOR EACH SIMULATED EEG - DISTRIBUTION COMBINATION . SAMPLE SIZE IS 100,000 AND 100 INTERVALS USED FOR HISTOGRAM. A NORMAL PROBABILITY DENSITY CURVE IS SUPERIMPOSED FOR COMPARISON.

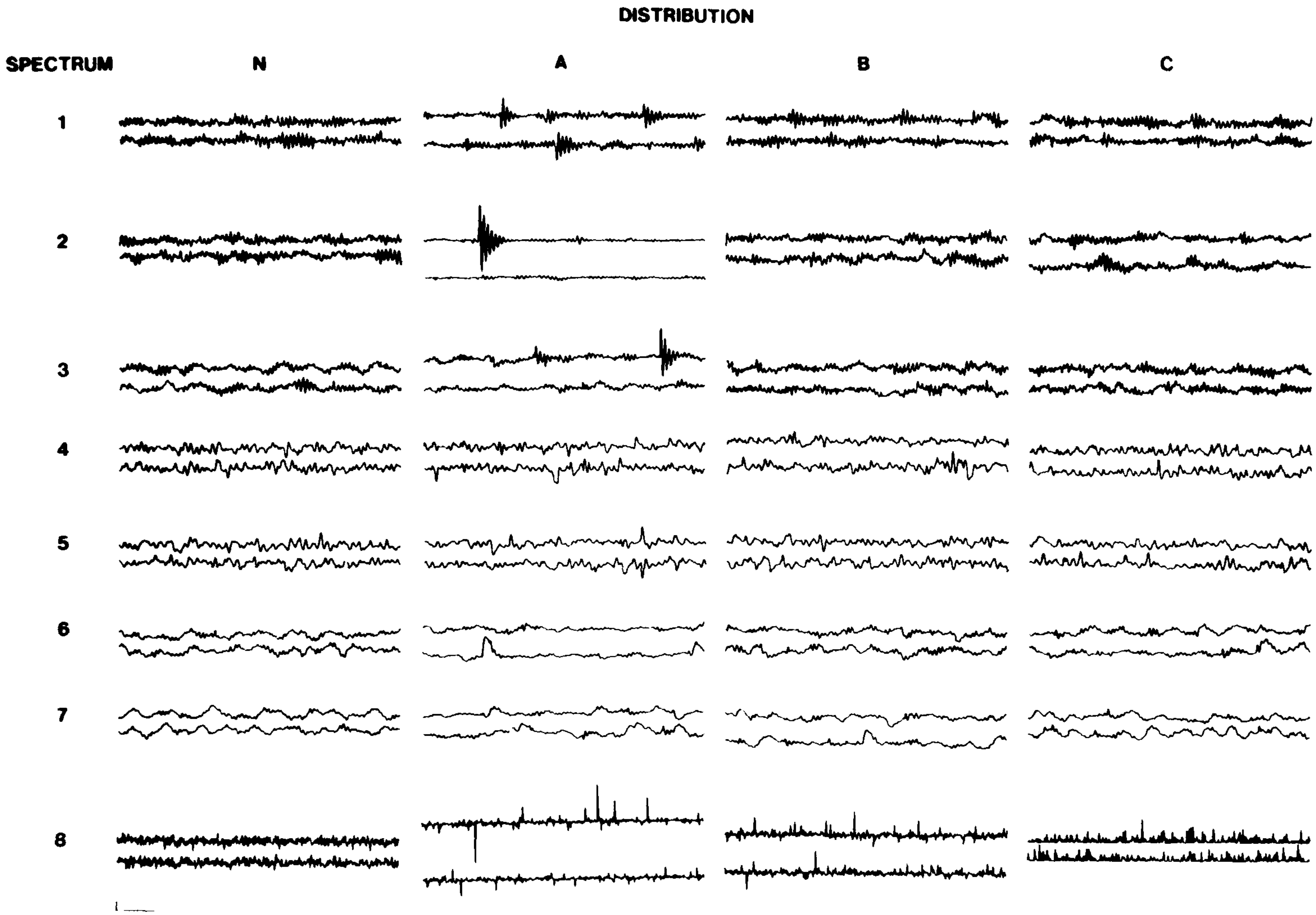

FIGURE 3 SAMPLE TIME SERIES FOR ALL SPECTRUM/DISTRIBUTION COMBINATIONS. TIME MARKER IS ONE SECOND. SEE TEXT FOR DETAILS.

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